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# Models of Neutrino Masses and Baryogenesis

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## Abstract

Majorana masses of the neutrino implies lepton number violation and is intimately related to the lepton asymmetry of the universe, which gets related to the baryon asymmetry of the universe in the presence of the sphalerons during the electroweak phase transition. Assuming that the baryon asymmetry of the universe is generated before the electroweak phase transition, it is possible to discriminate different classes of models of neutrino masses. While see-saw mechanism and the triplet higgs mechanism are preferred, the Zee-type radiative models and the R-parity breaking models requires additional inputs to generate baryon asymmetry of the universe during the electroweak phase transition.

# 1 Introduction

Two important issues of lepton number violation are intimately related to each other. One is the possible existence of neutrino Majorana masses, as evidenced by the ongoing excitement generated by the recent report of atmospheric neutrino oscillations [1], as well as previous other indications of solar [2] and accelerator [3] neutrino oscillations. The other is one of the very challenging question in cosmology to generate the baryon asymmetry of the universe starting from a symmetric universe [4]. Since the electroweak anomalous processes breaks both the baryon and the lepton numbers, still conserving the  $(B - L)$  quantum number, the baryon asymmetry of the universe is no longer independent of the lepton number violation of the universe [5, 6, 7, 8]. If there is very fast lepton number violation before the electroweak phase transition, then that can erase the  $(B - L)$  asymmetry of the universe [6] and hence the baryon asymmetry of the universe. On the other hand, if any lepton asymmetry is generated at some high temperature, that can get converted to a baryon asymmetry of the universe before and during the electroweak phase transition [7].

Lepton number violation is required to give a Majorana mass to the neutrinos. Depending on the scale at which this lepton number is violated, this interaction may or may not satisfy the out-of-equilibrium condition. If this interaction is faster than the expansion rate of the universe, it can erase all lepton asymmetry of the universe before the electroweak phase transition. In those models one then require additional inputs to explain the baryon asymmetry of the universe. On the other hand, in models of leptogenesis the lepton number violating interaction required to give Majorana masses to the neutrino also satisfy the out-of-equilibrium condition. If there is enough CP violation in the leptonic sector [9], then this can generate a  $(B - L)$  asymmetry of the universe. The anomalous baryon number violation in the presence of the sphalerons will then convert this  $(B - L)$  asymmetry to a baryon asymmetry of the universe before the electroweak phase transition [5, 8]. It is also possible to generate a baryon asymmetry of the universe during the electroweak phase transition [10], but the condition that this asymmetry will not be erased after the electroweak phase transition gives a strong bound on the mass of the higgs [11], which makes these less likely. As a result leptogenesis appears to be the most attractive scenario for generating a baryon asymmetry of the universe at present. In this article we shall thus summarise the possibility

of leptogenesis in different models of neutrino masses.

We shall first review the original idea of baryogenesis in the context of grand unified theories and show why  $(B - L)$  is always conserved in the baryon asymmetry thus generated. Then we show the relationship between the baryon and lepton number in the presence of the sphaleron processes and how  $(B + L)$  is washed out. This also implies constraints on the lepton number violation and hence on the neutrino masses. At the end we discuss different classes of models of neutrino masses which can naturally accomodate leptogenesis and then summarise.

## 2 GUT baryogenesis

The subject of baryogenesis originated when Sakharov [12] pointed out that for the generation of a baryon asymmetry of the universe we need three conditions

- (A) *Baryon number violation,*
- (B)  *$C$  and  $CP$  violation,* and
- (C) *Departure from thermal equilibrium.*

It was then realised that grand unified theories (GUTs) satisfies all these criterion [4, 13, 14].

The quark-lepton unification implies baryon number violation in GUTs. Since fermions belong to chiral representation,  $C$  is maximally violated. Departure from thermal equilibrium was also naturally satisfied in these models since the scale of unification is sufficiently high, and the universe was expanding very fast in that epoch. So, any reasonable GUT coupling would imply departure from equilibrium. Violation of  $CP$  was then the only crucial point, which had to be incorporated in these theories. However, it was not difficult to consider some of the couplings to be complex so that there exist tree level and one loop diagrams which could interfere to give us enough baryon asymmetry in the decays of the heavy gauge and higgs bosons [14].

This was considered to be one of the major successes of GUTs that it can explain the baryon asymmetry of the universe. After several years it was realised that the chiral nature

of the weak interaction also breaks the global baryon and lepton numbers in the standard model [15]. Although both  $B$  and  $L$  are broken, a combination  $(B - L)$  remains invariant since the baryon and lepton number anomalies happens to be the same in the standard model. Since these classical global  $(B + L)$  number symmetry is broken by quantum effects due to the presence of the anomaly, these processes were found to be very weak at the zero temperature. But at finite temperature these  $(B + L)$  number violating interactions were found to be very strong in the presence of some static topological field configuration - sphalerons [5]. In fact, during the period

$$10^{12}\text{GeV} \gg T \gg 10^2\text{GeV}$$

these interactions are so strong that in no time the particles and anti-particles attain their equilibrium distributions. As a result, since  $CPT$  is conserved and hence the masses of the particles and anti-particles are same, the number density of baryons becomes same as that of the anti-baryons and that will wash out any primordial  $(B + L)$  asymmetry of the universe. We shall now discuss why GUT baryogenesis always generated only  $(B + L)$  asymmetry [16], which would be erased by the sphaleron transitions.

In specific GUT scenarios such as  $SU(5)$  and  $SO(10)$ ,  $(B - L)$  is either a global or a local symmetry respectively. Hence the asymmetry generated by the above mechanism is  $(B - L)$  conserving [14]. When the scalar or vector bosons decay only into fermions, any attempt to generate a  $(B - L)$  asymmetry leads to its large suppression in all these models. We shall now prove that if the decay products are SM fermions only, this is in fact a generic property of any baryon asymmetry generated by the above described mechanism. This follows from an operator analysis analogous to the one used to show that the minimal scenarios of proton decay conserve  $(B - L)$  [17]. For definiteness we consider scalars  $X$  and  $Y$ , but obviously the result generalizes also to vectors.

Baryogenesis is possible in GUTs because there exist new gauge and Higgs bosons, whose decays violate baryon number. When these heavy particles (say  $X$ ) decay into two quarks and into a quark and an antilepton, the baryon and lepton numbers are broken [4]. For  $CP$  violation this mechanism requires two heavy gauge or Higgs bosons,  $X$  and  $Y$ , each of which

should have two decay modes,

$$\begin{aligned} X &\rightarrow A + B^*, & \text{and} & & X &\rightarrow C + D^*, \\ Y &\rightarrow A + C^*, & \text{and} & & Y &\rightarrow B + D^*, \end{aligned}$$

so that there exist one-loop vertex corrections to these decays. The required  $CP$  violation occurs due to the interference between tree and loop diagrams. As required by the out-of-equilibrium condition, masses of these particles must satisfy

$$\Gamma_X < H = 1.7\sqrt{g_*}\frac{T^2}{M_P} \quad \text{at } T = M_X, \quad (1)$$

where,  $\Gamma_X$  is the decay rate of the heavy particle  $X$ ;  $H$  is the Hubble constant;  $g_*$  is the effective number of massless degrees of freedom; and  $M_P$  is the Planck scale.

Let us start from the Lagrangian giving the decays of  $X$  and  $Y$ ,

$$\mathcal{L} = f_x^{ab}\bar{A}BX + f_x^{cd}\bar{C}DX + f_y^{ac}\bar{A}CY + f_y^{bd}\bar{B}DY, \quad (2)$$

where  $A, B, C, D$  denote any SM fermion. To obtain a nonzero  $CP$  violation from the interference between tree and vertex diagrams, we require  $X$  and  $Y$  to be distinct from each other and to have different decay modes. One can then write down all possible combinations of  $A, B, C$ , and  $D$ , with  $X$  and  $Y$ , and find out the decay modes of  $X$  and  $Y$ . Since the out-of-equilibrium condition and the nonvanishing of the absorptive part of the loop integral require these scalars  $X$  and  $Y$  to be much heavier than the fermions, we can integrate them out and write down the diagrams in terms of the four-fermion effective operators of the SM, as shown in Fig. 1. One can in principle also have the self-energy-type diagrams with the fermions in the loop for generating the  $CP$  asymmetry. In this case, after integrating out the heavy scalars, the effective diagrams in terms of the four-fermion operators are exactly the same as in the vertex-correction case, so the conclusions will not be changed.

This simple but crucial step allows us to use existing knowledge on SM four-fermion operators for baryon number violation which have been studied extensively in the literature [17]. It was found that all these operators conserve  $(B - L)$  to the lowest order. Any  $(B - L)$  violating operator will be suppressed by  $\langle\phi\rangle^2/M_{GUT}^2$  compared to the  $(B + L)$  violating operators. In models with an intermediate symmetry breaking scale or with new

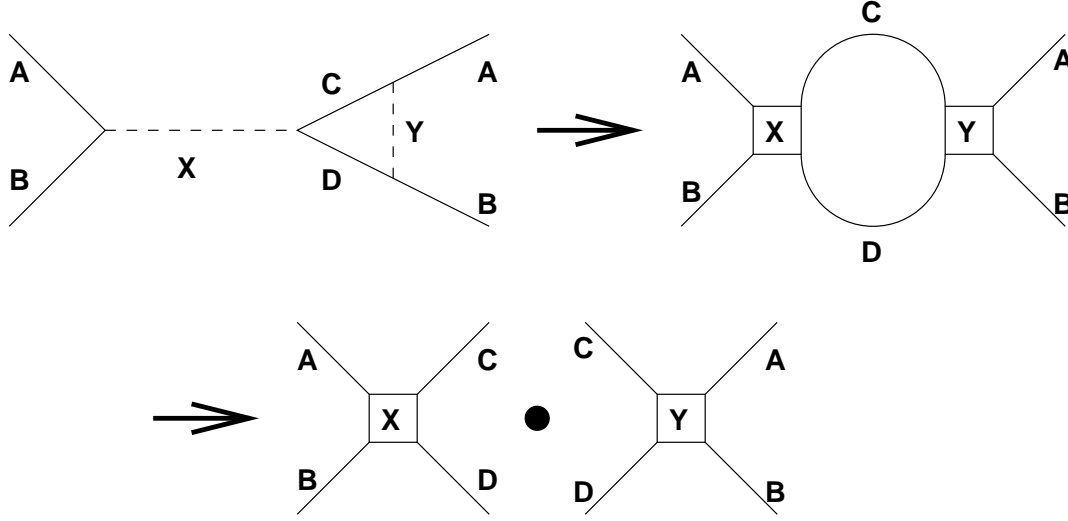


Figure 1: Interference of effective four-fermion operators which generates baryon asymmetry.

Higgs scalars at some intermediate scales, this suppression factor may be softened a little, but still strong enough to rule out any possibility of generating enough  $(B - L)$  asymmetry of the universe. On the other hand, any four-fermion operator which violates only lepton number requires all the fermions to be the same; hence it cannot generate the required  $CP$  asymmetry. Therefore a  $(B - L)$  asymmetry, needed to survive the sphaleron processes, is impossible to generate with the SM four-fermion operators.

### 3 Sphaleron processes in thermal equilibrium and relation between baryon and lepton numbers

Anomaly breaks any classical symmetry of the lagrangian at the quantum level. So, all local gauge theories should be free of anomalies. However, there may be anomalies corresponding to any global current, which means that such global symmetries of the classical lagrangian are broken through quantum effects. In the standard model the baryon and lepton number

global symmetries are anomalous [15]

$$\delta_\mu j_{(B+L)}^{\mu 5} = 6 \left[ \frac{\alpha_2}{8\pi} W_a^{\mu\nu} \tilde{W}_{a\mu\nu} + \frac{\alpha_1}{8\pi} Y^{\mu\nu} \tilde{Y}_{\mu\nu} \right]$$

which will break the  $(B + L)$  symmetry. However, the anomaly corresponding to the baryon and lepton numbers are same and as a result there is no anomaly for the  $(B - L)$  charge. Because of the anomaly [15],  $(B + L)$  is broken during the electroweak phase transition, but their rate is very small at zero temperature, since they are suppressed by quantum tunnelling probability,  $\exp[-\frac{2\pi}{\alpha_2}\nu]$ .

At finite temperature, this  $(B + L)$  number violation becomes very fast in the presence of a non-trivial static topological soliton configuration, called the sphalerons [5], and the quantum tunnelling suppression factor is now replaced by the Boltzmann factor  $\exp[-\frac{V_0}{T}\nu]$  where the potential or the free energy  $V_0$  is related to the mass of the sphaleron field. As a result, at temperatures between  $10^{12} GeV > T > 10^2 GeV$  the sphaleron mediated baryon and lepton number violating processes are in equilibrium. For the simplest scenario of  $\nu = 1$ , the sphaleron induced processes are  $\Delta B = \Delta L = 3$ , given by,

$$|vac\rangle \longrightarrow [u_L u_L d_L e_L^- + c_L c_L s_L \mu_L^- + t_L t_L b_L \tau_L^-]. \quad (3)$$

It can be shown that any  $(B - L)$  asymmetry before the electroweak phase transition will get converted to a baryon and lepton asymmetry of the universe, which can be seen from an analysis of the chemical potential [8].

Above the electroweak scale, all the particles could be assumed to be ultrarelativistic. The particle asymmetry, *i.e.* the difference between the number of particles ( $n_+$ ) and the number of antiparticles ( $n_-$ ) can be given in terms of the chemical potential of the particle species  $\mu$  (for antiparticles the chemical potential is  $-\mu$ ) as

$$n_+ - n_- = n_d \frac{gT^3}{6} \left( \frac{\mu}{T} \right), \quad (4)$$

where  $n_d = 2$  for bosons and  $n_d = 1$  for fermions.

In the standard model there are quarks and leptons  $q_{iL}, u_{iR}, d_{iR}, l_{iL}$  and  $e_{iR}$ ; where,  $i = 1, 2, 3$  corresponds to three generations. In addition, the scalar sector consists of the usual Higgs doublet  $\phi$ , which breaks the electroweak gauge symmetry  $SU(2)_L \times U(1)_Y$  down

to  $U(1)_{em}$ . In Table 1, we presented the relevant interactions and the corresponding relations between the chemical potentials. In the third column we give the chemical potential which we eliminate using the given relation. We start with chemical potentials of all the quarks ( $\mu_{uL}, \mu_{dL}, \mu_{uR}, \mu_{dR}$ ); leptons ( $\mu_{aL}, \mu_{\nu aL}, \mu_{aR}$ , where  $a = e, \mu, \tau$ ); gauge bosons ( $\mu_W$  for  $W^-$ , and 0 for all others); and the Higgs scalars ( $\mu_-^\phi, \mu_0^\phi$ ).

Table 1: Relations among the chemical potentials

Interactions	$\mu$ relations	$\mu$ eliminated
$D_\mu \phi^\dagger D_\mu \phi$	$\mu_W = \mu_-^\phi + \mu_0^\phi$	$\mu_-^\phi$
$\overline{q_L} \gamma_\mu q_L W^\mu$	$\mu_{dL} = \mu_{uL} + \mu_W$	$\mu_{dL}$
$\overline{l_L} \gamma_\mu l_L W^\mu$	$\mu_{iL} = \mu_{\nu iL} + \mu_W$	$\mu_{iL}$
$\overline{q_L} u_R \phi^\dagger$	$\mu_{uR} = \mu_0 + \mu_{uL}$	$\mu_{uR}$
$\overline{q_L} d_R \phi$	$\mu_{dR} = -\mu_0 + \mu_{dL}$	$\mu_{dR}$
$\overline{l_L} e_R \phi$	$\mu_{iR} = -\mu_0 + \mu_{iL}$	$\mu_{iR}$

The chemical potentials of the neutrinos always enter as a sum and for that reason we can consider it as one parameter. We can then express all the chemical potentials in terms of the following independent chemical potentials only,  $\mu_0 = \mu_0^\phi$ ;  $\mu_W$ ;  $\mu_u = \mu_{uL}$ ;  $\mu = \sum_i \mu_i = \sum_i \mu_{\nu iL}$ . We can further eliminate one of these four potentials by making use of the relation given by the sphaleron processes,  $3\mu_u + 2\mu_W + \mu = 0$ . We then express the baryon number, lepton numbers and the electric charge and the hypercharge number densities in terms of these independent chemical potentials,

$$\begin{aligned}
B &= 12\mu_u + 6\mu_W; & L_i &= 3\mu + 2\mu_W - \mu_0 \\
Q &= 24\mu_u + (12 + 2m)\mu_0 - (4 + 2m)\mu_W; & Q_3 &= -(10 + m)\mu_W
\end{aligned}$$

where  $m$  is the number of Higgs doublets  $\phi$ .

At temperatures above the electroweak phase transition,  $T > T_c$ , both  $\langle Q \rangle$  and  $\langle Q_3 \rangle$  must vanish, while below the critical temperature  $\langle Q \rangle$  should vanish, but since  $SU(2)_L$  is now broken we can consider  $\mu_0^\phi = 0$  and  $Q_3 \neq 0$ . These conditions and the sphaleron induced  $B - L$  conserving,  $B + L$  violating condition will allow us to write down the baryon

asymmetry in terms of the  $B - L$  number density as,

$$B(T > T_c) = \frac{24 + 4m}{66 + 13m} (B - L) \quad B(T < T_c) = \frac{32 + 4m}{98 + 13m} (B - L). \quad (5)$$

Thus the baryon asymmetry of the universe after the electroweak phase transition will depend only on the primordial  $(B - L)$  asymmetry of the universe, while all the primordial  $(B + L)$  asymmetry will be washed out.

Before proceeding further, we shall briefly discuss what do we mean when we say that some interaction is fast and that will erase some asymmetry [4, 12, 18]. In equilibrium the number density of particles with non-zero charge  $Q$  would be same as the antiparticle number density since the expectation value of the conserved charge vanishes. A mathematical formulation of this statement reads that the expectation value of any conserved charge  $Q$  is given by,

$$\langle Q \rangle = \frac{\text{Tr} [Q e^{-\beta H}]}{\text{Tr} [e^{-\beta H}]}$$

and since any conserved charge  $Q$  is odd while  $H$  is even under  $CPT$  transformation this expectation value vanishes. So for the generation of the baryon asymmetry of the universe we have to circumvent this theorem either by including nonzero chemical potential, or go away from equilibrium or violate  $CPT$ . In most of the popular models  $CPT$  conservation is assumed and one starts with vanishing chemical potential for all the fields which ensures that the entropy is maximum in chemical equilibrium. Then to generate the baryon asymmetry of the universe one needs to satisfy the out-of-equilibrium condition [4, 12, 18].

The requirement for the out-of-equilibrium condition may also be stated in a different way [4]. If we assume that the chemical potential associated with  $B$  is zero and  $CPT$  is conserved, then in thermal equilibrium the phase space density of baryons and antibaryons, given by  $[1 + \exp(\sqrt{p^2 + m^2}/kT)]^{-1}$  are identical and hence there cannot be any baryon asymmetry.

Whether a system is in equilibrium or not can be understood by solving the Boltzmann equations. But a crude way to put the out-of-equilibrium condition is to say that the universe expands faster than some interaction rate. For example, if some  $B$ -violating interaction is slower than the expansion rate of the universe, this interaction may not bring the distribution of baryons and antibaryons of the universe in equilibrium. In other words, before the chemical

potentials of the two states gets equal, they move apart from each other. Thus we may state the out-of-equilibrium condition as

$$\Gamma < \sqrt{1.7g_*} \frac{T^2}{M_P} \quad (6)$$

where,  $\Gamma$  is the interaction rate under discussion,  $g_*$  is the effective number of degrees of freedom available at that temperature  $T$ , and  $M_P$  is the Planck scale.

## 4 Constraints on neutrino masses

In the standard model there is no lepton number violation. However, one can consider a higher dimensional effective operator which violates  $(B - L)$ , given by

$$L = \frac{2}{M} l_L l_L \phi \phi + h.c. \quad (7)$$

There is no origin of such interactions within the standard model. So one expects that some new interaction at some high energy will give us this effective interaction at low energy.

The scale of the new interaction  $M$ , which is also the scale of lepton number (and also  $(B - L)$  number) violation, will determine if this interaction is fast enough to erase all primordial  $(B - L)$  asymmetry. Since during the same time  $(B + L)$  asymmetry is also washed out by the sphaleron transitions, there will not be any residual baryon asymmetry of the universe after the electroweak phase transition. As a result, the survival of the baryon asymmetry of the universe will then require this interaction to be slower than the expansion rate of the universe,

$$\Gamma_{L \neq 0} \sim \frac{0.122}{\pi} \frac{T^3}{M^2} < 1.7 \sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T \sim 100 GeV \quad (8)$$

which gives a bound [6] on the lepton number violating scale to be,  $M > 10^9 GeV$ . When the higgs doublets  $\phi$  acquires a  $vev$ , the higher dimensional operator will induce a Majorana mass of the left-handed neutrinos. This bound on the heavy scale  $M$  will then imply a bound on the mass of the left-handed neutrinos,

$$m_\nu < 50 keV.$$

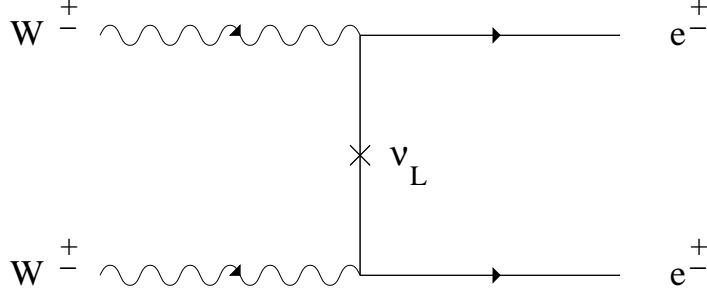


Figure 2: Lepton number violating processes  $W^\pm + W^\pm \rightarrow e^\pm + e^\pm$  mediated by the left handed Majorana neutrinos.

It is also possible to give a bound on the neutrino mass in a more general way [19]. Unless the neutrinos are Dirac particles [20], during the electroweak phase transition there will be interactions of the type,

$$W^+ + W^+ \rightarrow e_i^+ + e_j^+ \quad \text{and} \quad W^- + W^- \rightarrow e_i^- + e_j^- \quad (9)$$

which violate lepton number. These interactions are mediated by a virtual left-handed neutrino exchange as shown in figure 2. Here  $i$  and  $j$  are the generation indices. Depending on the physical mass of the left-handed Majorana neutrinos these processes can wash out any baryon asymmetry between the time when the higgs acquires a  $vev$  and the  $W^\pm$  freeze out, *i.e.*, between the energy scales 250 GeV and 80 GeV.

The condition that these processes will be slower than the expansion rate of the universe,

$$\Gamma(WW \rightarrow e_i e_j) = \frac{\alpha_W^2 (m_\nu)_{ij}^2 T^3}{m_W^4} < 1.7 \sqrt{g_*} \frac{T^2}{M_p} \quad \text{at } T = M_W \quad (10)$$

gives a bound on the Majorana mass of the left-handed neutrinos to be,

$$(m_\nu)_{ij} < 20 keV. \quad (11)$$

This bound is on each and every element of the mass matrix and not on the physical states. There are other lepton number violating interactions like the scattering processes  $\phi + \phi \rightarrow l_i + l_j$  (mediated by a virtual left-handed neutrino) and decays of  $W^\pm$  and the higgs  $\phi$ , which also give similar bounds on the left-handed neutrino mass.

In some specific models one may give stronger bounds on the mass of the neutrinos [21, 22]. In models with right handed neutrinos ( $N_{Ri}, i = e, \mu, \tau$ ), the neutrino masses comes from the see-saw mechanism [23]. The lagrangian for the lepton sector containing the mass terms of the singlet right handed neutrinos  $N_i$  and the Yukawa couplings of these fields with the light leptons is given by,

$$\mathcal{L}_{int} = M_i \overline{(N_{Ri})^c} N_{Ri} h_{\alpha i} \overline{\ell_{L\alpha}} \phi N_{Ri} \quad (12)$$

where  $\phi$  is the usual higgs doublet of the standard model;  $\ell_{L\alpha}$  are the light leptons,  $h_{\alpha i}$  are the complex Yukawa couplings and  $\alpha$  is the generation index. Without loss of generality we work in a basis in which the Majorana mass matrix of the right handed neutrinos is real and diagonal with eigenvalues  $M_i$ .

Once the higgs doublet  $\phi$  acquires a *vev*, the masses of the neutrinos in the basis  $[\nu_{L\alpha} \ N_{Ri}]$  is given by,

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \quad (13)$$

where,  $m \equiv h_{\alpha i} \langle \phi \rangle$  and  $M \equiv M_{ij}$  are  $3 \times 3$  matrices. In the limit when all eigenvalues of  $M$  are much heavier than those of  $m$ , and the matrix  $M$  is not singular, this matrix may be block diagonalised. It then gives three heavy right handed Majorana neutrinos with masses  $\sim M$  and the Majorana mass matrix of the left-handed neutrinos will be given by,

$$m_\nu = m \frac{1}{M} m^T. \quad (14)$$

In this scenario the see-saw masses of the left-handed neutrinos explain naturally why they are so light.

The decay of  $N_{Ri}$  into a lepton and an antilepton,

$$\begin{aligned} N_{Ri} &\rightarrow \ell_{jL} + \bar{\phi}, \\ &\rightarrow \ell_{jL}^c + \phi. \end{aligned} \quad (15)$$

breaks lepton number. Since the lightest of the right handed neutrinos (say  $N_1$ ) will decay at the end, this interaction ( $N_1$  decay) should be slow enough so as not to erase the baryon

asymmetry of the universe, which now implies

$$\frac{|h_{\alpha 1}|^2}{16\pi} M_1 < 1.7 \sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T = M_1 \quad (16)$$

which can then give a very strong bound [21, 22] on the mass of the lightest of the left-handed neutrinos to be

$$m_\nu < 4 \times 10^{-3} eV.$$

In models [24, 25, 26, 27, 28], where the left-handed neutrino mass is not related to any heavy neutrinos through see-saw mechanism, the abovementioned bounds may not be valid. In addition, there are several specific cases even within the framework of see-saw models (like the singular see-saw mechanism where  $\det M = 0$ ), where these bounds are not applicable. These bounds are also not valid if some global  $U(1)$  symmetry is exactly conserved up to an electroweak anomaly [29]. Furthermore, in some very specific models where a baryon asymmetry of the universe is generated after the electroweak phase transition [30], or there are some extra baryon number carrying singlets which decays after the electroweak phase transition [31], it is possible to avoid all the bounds from constraints of survival of the baryon asymmetry of the universe.

We shall now discuss similar bounds on the supersymmetric R-parity violating and Zee type radiative models. Although the earlier bounds on the see-saw mechanism is not applicable when the decays of the right handed neutrinos generate a lepton asymmetry of the universe, since leptogenesis is not possible in these R-parity breaking models or the Zee-type models, in these models one needs additional inputs to generate a baryon asymmetry of the universe.

In the R-parity violating models, the unavoidable lepton number violation at the supersymmetry breaking scale will erase any primordial  $B$  or  $L$  or  $B-L$  asymmetry [22, 32, 16, 33]. This is so unless  $B - 3L_i$  is conserved [29, 34] even after the electroweak phase transition. This has been pointed out earlier from a general dimensional analysis, but none of the existing models of neutrino masses through R-parity violation could accommodate this symmetry since that cannot allow required neutrino mixing matrix. Possible solutions to this problem could be to break R-parity spontaneously after the electroweak symmetry breaking [35], or to generate a baryon asymmetry of the universe in R-parity breaking scenarios [36], or gen-

erate a baryon asymmetry of the universe during the electroweak phase transition [10]. But these models are incapable of accomodating the interesting feature of leptogenesis, namely generating a baryon asymmetry of the universe from the interaction which gives a neutrino mass.

Similarly the Zee-type models [25] considered so far cannot account for the observed baryon asymmetry of the universe, if they have to explain the present neutrino mass spectrum. Although the radiative models have the advantage that they can reproduce the required maximal mixing naturally, they erase any primordial lepton asymmetry of the universe and hence the baryon asymmetry of the universe. This severe constraint on the Zee-type models are also valid in both supersymmetric and non-supersymmetric scenarios. It is not impossible to find an alternative where a baryon asymmetry of the universe is generated after it has been washed out by this interaction, but that will not be related to the neutrino mass.

In the MSSM, R-parity of a particle is defined as

$$R \equiv (-1)^{3B+L+2J}, \quad (17)$$

where  $B$  is its baryon number,  $L$  its lepton number, and  $J$  its spin angular momentum. Hence the SM particles have  $R = +1$  and their supersymmetric partners have  $R = -1$ . Using the common notation where all chiral superfields are considered left-handed, the three families of leptons and quarks are given by

$$L_i = (\nu_i, e_i) \sim (1, 2, -1/2), \quad e_i^c \sim (1, 1, 1), \quad (18)$$

$$Q_i = (u_i, d_i) \sim (3, 2, 1/6), \quad u_i^c \sim (3^*, 1, -2/3), \quad d_i^c \sim (3^*, 1, 1/3), \quad (19)$$

where  $i$  is the family index, and the two Higgs doublets are given by

$$H_1 = (h_1^0, h_1^-) \sim (1, 2, -1/2), \quad H_2 = (h_2^+, h_2^0) \sim (1, 2, 1/2), \quad (20)$$

where the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  content of each superfield is also indicated. If R-parity is conserved, the superpotential is restricted to have only the terms

$$W = \mu H_1 H_2 + f_{ij}^e H_1 L_i e_j^c + f_{ij}^d H_1 Q_i d_j^c + f_{ij}^u H_2 Q_i u_j^c. \quad (21)$$

If R-parity is violated but not baryon number, then the superpotential contains the additional terms

$$W' = \epsilon_i L_i H_2 + \lambda_{ijk} L_i L_j e_k^c + \lambda'_{ijk} L_i Q_j d_k^c, \quad (22)$$

resulting in nonzero neutrino masses either from mixing with the neutralino mass matrix [37] or in one-loop order [38].

If lepton-number violating interactions such as

$$L_i + Q_j \rightarrow (\tilde{d}_k^c)^* \rightarrow H_1 + Q_l \quad (23)$$

are in equilibrium in the early universe, any pre-existing lepton asymmetry would be erased. To make sure that this does not happen, the following condition has to be satisfied:

$$\frac{\lambda'^2 T}{8\pi} \lesssim 1.7 \sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T = M_{SUSY}. \quad (24)$$

Assuming that the supersymmetry breaking scale  $M_{SUSY}$  is  $10^3$  GeV, we find

$$\lambda' \lesssim 2 \times 10^{-7}, \quad (25)$$

which is very much below the typical minimum value of  $10^{-4}$  needed for radiative neutrino masses [26]. A similar bound was presented from dimensional arguments [22, 32]. Larger values of  $\lambda'$  are allowed if there is a conserved  $(B - 3L_i)$  symmetry [29]. However, there would be other severe phenomenological restrictions in that case [34]. This bound cannot be evaded even if one uses the bilinear term for neutrino masses instead, because the induced mixing would introduce trilinear couplings which violate lepton number and an effective  $\lambda'$  is unavoidable. This means that although R-parity violation may exist, it will have very little consequences. In particular, it will not contribute significantly to neutrino masses.

In models of radiative neutrino masses [25], in addition to the suppression due to the  $1/16\pi^2$  factor of each loop, there is often another source of suppression due to the Yukawa couplings involved. In the original Zee model, the SM is extended to include a charged scalar  $\chi^+$  and a second Higgs doublet.

The relevant terms of the interaction Lagrangian are given by

$$\mathcal{L} = \sum_{i < j} f_{ij} (\nu_i e_j - e_i \nu_j) \chi^+ + \mu (\phi_1^+ \phi_2^0 - \phi_1^0 \phi_2^+) \chi^- + H.c., \quad (26)$$

where two Higgs doublets are needed or else there would be no  $\phi\phi\chi$  coupling. Lepton number is violated in the above by two units, hence we expect the realization of an effective dimension-five operator  $\Lambda^{-1}\phi^0\phi^0\nu_i\nu_j$  for naturally small Majorana neutrino masses [39]. This occurs here in one loop and the elements of the  $3 \times 3$  neutrino mass matrix are given by

$$(m_\nu)_{ij} = f_{ij}(m_i^2 - m_j^2) \left( \frac{\mu v_2}{v_1} \right) F(m_\chi^2, m_{\phi_1}^2), \quad (27)$$

where  $v_{1,2} \equiv \langle \phi_{1,2}^0 \rangle$  and  $m_i$  are the charged-lepton masses which come from  $\phi_1$  but not  $\phi_2$ . The function  $F$  is given by

$$F(m_1^2, m_2^2) = \frac{1}{16\pi^2} \frac{1}{m_1^2 - m_2^2} \ln \frac{m_1^2}{m_2^2}. \quad (28)$$

Since the  $m_\tau^2$  terms in Eq. (11) are likely to be dominant, this model has two nearly mass-degenerate neutrinos which mix maximally. This is very suitable for explaining the atmospheric neutrino data [1], but only in conjunction with the LSND data [3]. Let  $m_\chi = 1$  TeV,  $m_{\phi_1} = 100$  GeV,  $\mu = 100$  GeV,  $v_2/v_1 = 1$ , and  $f_{\mu\tau} = f_{e\tau} = 10^{-7}$  to satisfy Eq. (9), then the  $m_\tau^2$  terms generate a neutrino mass of 0.0013 eV, which is very much below the necessary 1 eV or so indicated by the LSND data. We note that Eq. (8) constrains the combination  $f^2/m_\chi$ , whereas  $m_\nu$  goes like  $f/m_\chi^2$ . Hence neutrino masses would only decrease if we increase  $m_\chi$ . As long as there is a suppression from  $m_\tau^2$  (which comes of course from the Yukawa coupling  $m_\tau/v_1$ ), the conflict with leptogenesis is a real problem.

## 5 Models of Leptogenesis

In the standard model neutrinos are massless. To make them massive, there exist four generic mechanisms, namely, the see-saw mechanism [23], the triplet higgs mechanism [27, 28], the radiative mass generation [25] and through R-parity violation [37]. All these models require lepton number violation, which can erase the primordial baryon asymmetry of the universe. So, the most promising scenario will be to see if this lepton number violation could be used to generate a baryon asymmetry of the universe. This is done in models of leptogenesis [7, 27]. The see-saw mechanism and the triplet higgs mechanism for neutrino masses are the two mechanisms, which can accomodate leptogenesis in the minimal models, which we shall

summarise next. Although one can extend the Zee-type models also or have a complicated scenario of R-parity breaking models for generating a lepton asymmetry of the universe [36], we shall not discuss them in this article.

## 5.1 Leptogenesis with right-handed neutrinos

To give a small Majorana mass to the left-handed neutrino through a see-saw mechanism, right-handed neutrinos were introduced ( $N_{Ri}, i = e, \mu, \tau$ ). In these models neutrino masses come from the see-saw mechanism [23]. The lagrangian for the lepton sector containing the mass terms of the singlet right handed neutrinos  $N_i$  and the Yukawa couplings of these fields with the light leptons is given by eqn (15). Without loss of generality we work in a basis in which the Majorana mass matrix of the right handed neutrinos is real and diagonal with eigenvalues  $M_i$ , and assume  $M_3 > M_2 > M_1$ .

Because of the Majorana mass term, the decay of  $N_{Ri}$  into a lepton and an antilepton, breaks lepton number. There are two sources of CP violation in this scenario :

- (i) vertex type one loop diagrams which interferes with the tree level diagram given by figure 3. This is similar to the  $CP$  violation coming from the penguin diagram in  $K$ -decays.
- (ii) self energy type one loop diagrams could interfere with the tree level diagrams to produce CP violation as shown in figure 4. This is similar to the  $CP$  violation in  $K - \bar{K}$  oscillation, entering in the mass matrix of the heavy Majorana neutrinos.

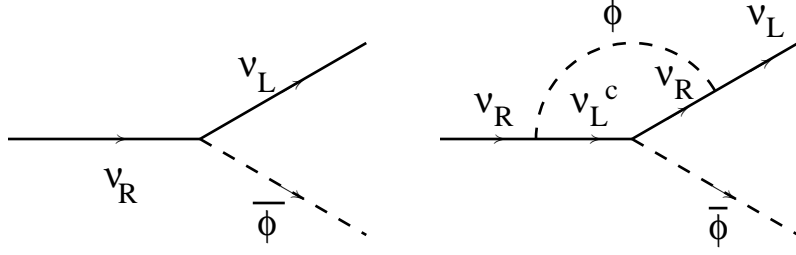


Figure 3: Tree and one loop vertex correction diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

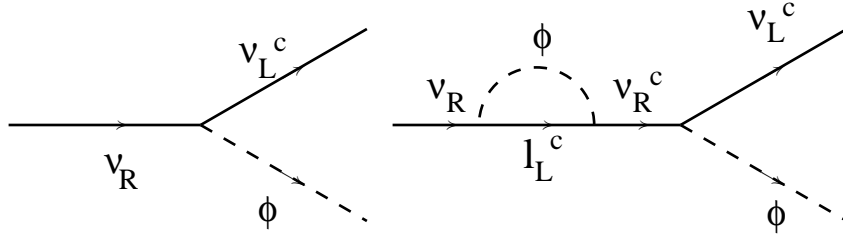


Figure 4: Tree and one loop self energy diagrams contributing to the generation of lepton asymmetry in models with right handed neutrinos

In the first paper on leptogenesis [6], the vertex type diagram was only mentioned. Subsequently, it has been extensively studied [40] and the amount of  $CP$  asymmetry is calculated to be,

$$\delta = -\frac{1}{8\pi} \frac{M_1 M_2}{M_2^2 - M_1^2} \frac{\text{Im}[\sum_{\alpha} (h_{\alpha 1}^* h_{\alpha 2}) \sum_{\beta} (h_{\beta 1}^* h_{\beta 2})]}{\sum_{\alpha} |h_{\alpha 1}|^2} \quad (29)$$

In this expression it has been assumed that the main contribution to the asymmetry comes from the lightest right handed neutrino ( $N_1$ ) decay, when the other heavy neutrinos have already decayed away.

Initially the self energy diagram was considered for  $CP$  violation as an additional contribution [41]. It was then pointed out [42] that this  $CP$  violation enters in the mass matrix as in the  $K - \bar{K}$  oscillation. Before they decay, the right handed neutrinos were considered to oscillate to an anti-neutrino and since the rate of  $particle \rightarrow anti - particle \neq anti - particle \rightarrow particle$ , an asymmetry in the right handed neutrino was obtained before they decay [43]. As a result, when the two heavy right handed neutrinos are almost degenerate, *i.e.*, the mass difference is comparable to their width, there may be a resonance effect which can enhance the  $CP$  asymmetry by few orders of magnitude [44]. This effect was then confirmed by other calculations [45, 46], one of which [45] uses a field-theoretic resummation approach [47] used earlier to treat unstable intermediate states. For large mass difference the amount of  $CP$  asymmetry from the self energy contribution becomes equal to the vertex correction, which has to be added to get the final asymmetry.

Although the  $CP$  asymmetry was found to be non-vanishing, in thermal equilibrium unitarity and  $CPT$  would mean that there is no asymmetry in the final decay product. However, when the out-of-equilibrium condition of the heavy neutrinos decay is considered properly, one could get an asymmetry as expected. Consider the decays of  $K_L$  and  $K_S$ . If they were generated in the early universe, in a short time scale  $K_S$  could decay and recombine, but  $K_L$  may not be able to decay or recombine. As a result in the decay product there will be an asymmetry in  $K$  and  $\bar{K}$  if there is  $CP$  violation. In the lepton number violating two body scattering processes  $CP$  violation in the real intermediate state plays the most crucial role, which comes since the decay take place away from thermal equilibrium.

In the case of right handed neutrino decay, the asymmetry is generated when the lightest one (say  $N_1$ ) decay. Before its decay, the pre-existing lepton asymmetry is washed out by its lepton number violating interactions. So the out-of-equilibrium condition now implies that the lightest right-handed neutrino should satisfy the out-of-equilibrium condition when it decays, which is given by,

$$\frac{|h_{\alpha 1}|^2}{16\pi} M_1 < 1.7\sqrt{g_*} \frac{T^2}{M_P} \quad \text{at } T = M_1 \quad (30)$$

which gives a bound on the mass of the lightest right-handed neutrino to be  $m_{N_1} < 10^7 GeV$ . Finally the lepton asymmetry and hence a  $(B - L)$  asymmetry generated at this scale gets

converted to a baryon asymmetry of the universe in the presence of sphaleron induced processes.

## 5.2 Leptogenesis with triplet higgs

To give a Majorana mass to the neutrino, one can either introduce a right handed neutrino as in the see-saw mechanism, or else one can introduce two complex  $SU(2)_L$  triplet higgs [28, 27] scalars ( $\xi_a \equiv (1, 3, -1); a = 1, 2$ ). The *vevs* of the triplet higgses can give small Majorana masses to the neutrinos through the interaction

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+(\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + h.c. \quad (31)$$

If the triplet higgs acquires a *vev* and break lepton number spontaneously, then there will be Majorons in the problem which is ruled out by precision Z-width measurement at LEP. However, in a variant of this model [27] lepton number is broken explicitly through an interaction of the triplet with the higgs doublet

$$V = \mu(\bar{\xi}^0 \phi^0 \phi^0 + \sqrt{2} \xi^- \phi^+ \phi^0 + \xi^{--} \phi^+ \phi^+) + h.c. \quad (32)$$

Let  $\langle \phi^0 \rangle = v$  and  $\langle \xi^0 \rangle = u$ , then the conditions for the minimum of the potential relates the *vev* of the two scalars by  $u \simeq \frac{-\mu v^2}{M^2}$ , where  $M$  is the mass of the triplet higgs scalar and the neutrino mass matrix becomes  $-2f_{ij}\mu v^2/M^2 = 2f_{ij}u$ .

In this case the lepton number violation comes from the decays of the triplet higgs  $\xi_a$ ,

$$\xi_a^{++} \rightarrow \begin{cases} l_i^+ l_j^+ & (L = -2) \\ \phi^+ \phi^+ & (L = 0) \end{cases} \quad (33)$$

The coexistence of the above two types of final states indicates the nonconservation of lepton number. On the other hand, any lepton asymmetry generated by  $\xi_a^{++}$  would be neutralized by the decays of  $\xi_a^{--}$ , unless CP conservation is also violated and the decays are out of thermal equilibrium in the early universe. In this case there are no vertex corrections which can introduce CP violation. The only source of CP violation is the self energy diagrams of figure 5.

If there is only one  $\xi$ , then the relative phase between any  $f_{ij}$  and  $\mu$  can be chosen real. Hence a lepton asymmetry cannot be generated. With two  $\xi$ 's, even if there is only one

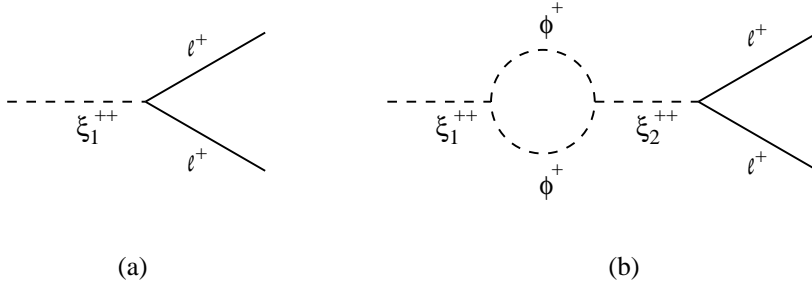


Figure 5: The decay of  $\xi_1^{++} \rightarrow l^+ l^+$  at tree level (a) and in one-loop order (b). A lepton asymmetry is generated by their interference in the triplet higgs model for neutrino masses.

lepton family, one relative phase must remain. As for the possible relative phases among the  $f_{ij}$ 's, they cannot generate a lepton asymmetry because they all refer to final states of the same lepton number.

In the presence of the one loop diagram, the mass matrix  $M_a^2$  and  $M_a^{*2}$  becomes different. This implies that the rate of  $\xi_b \rightarrow \xi_a$  no longer remains to be same as  $\xi_b^* \rightarrow \xi_a^*$ . Since by *CPT* theorem  $\xi_b^* \rightarrow \xi_a^* \equiv \xi_a \rightarrow \xi_b$ , what it means is that now  $\Gamma[\xi_a \rightarrow \xi_b] \neq \Gamma[\xi_b \rightarrow \xi_a]$ . This is a different kind of CP violation compared to the CP violation in models with right handed neutrinos. If we consider that the  $\xi_2$  is heavier than  $\xi_1$ , then after  $\xi_2$  decayed out the decay of  $\xi_1$  will generate an lepton asymmetry given by,

$$\delta \simeq \frac{\text{Im} \left[ \mu_1 \mu_2^* \sum_{k,l} f_{1kl} f_{2kl}^* \right]}{8\pi^2 (M_1^2 - M_2^2)} \left[ \frac{M_1}{\Gamma_1} \right]. \quad (34)$$

In this model the out-of-equilibrium condition is satisfied when the masses of the triplet higgs scalars are heavier than  $10^{13}$  GeV.

The lepton asymmetry thus generated after the Higgs triplets decayed away would be the same as the  $(B - L)$  asymmetry before the electroweak phase transition. During the electroweak phase transition, the presence of sphaleron fields would relate this  $(B - L)$  asymmetry to the baryon asymmetry of the universe. The final baryon asymmetry thus generated can then be given by the approximate relation  $\frac{n_B}{s} \sim \frac{\delta_2}{3g_* K (\ln K)^{0.6}}$ . This allows us

to obtain a neutrino mass of order eV or less, as well as the observed baryon asymmetry of the universe  $n_B/s \sim 10^{-10}$  as desired.

In general, it is not possible to discriminate the see-saw mechanism from the triplet higgs mechanism for neutrino masses. However, in some specific supersymmetric inflationary models, where the reheat temperature is lower than  $10^{10}$  GeV, the see-saw mechanism is preferred. On the other hand the leptogenesis scenario in the triplet higgs mechanism has several nice features, like the absence of the vertex diagrams or its detectability in the near future in the accelerators [27]. However, in the left-right symmetric models both the scenarios are present and can contribute to the neutrino masses as well as to leptogenesis [48].

## 6 Summary

The Majorana masses of the neutrinos implies lepton number violation. One very important consequence of this lepton number violation in the early universe is that it can erase any primordial baryon asymmetry of the universe in the presence of the sphaleron field before the electroweak phase transition. This gives bound on the mass on the neutrinos. While a general analysis can give somewhat weak bound, in some specific models these bounds could be very important. For example, in Zee-type radiative models or the R-parity breaking supersymmetric models this is very restrictive. If one attempts to explain the atmospheric neutrino problem, then these models would wash out all primordial baryon asymmetry of the universe. This implies that most models of neutrino masses based on these two scenarios are incomplete and more inputs are required to explain the present baryon asymmetry of the universe in these models. On the other hand, in the see-saw mechanism and the triplet higgs mechanism, the lepton number violation that gives masses to the neutrinos also generate a lepton asymmetry of the universe, which then get converted to a baryon asymmetry of the universe in the presence of the sphaleron field.

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